Studying the anisotropy of the gravitational wave stochastic background with LISA

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A plethora of gravitational wave stochastic backgrounds populate the sensitivity window of the Laser Interferometer Space Antenna. We show that LISA can detect the anisotropy of the background corresponding to the multipole moments of order $l = 2$ and 4. The signal-to-noise ratio generated by galactic white dwarf binary systems could be as high as ≈ 60 for 3 yrs of integration, and LISA could provide valuable information on the spatial distribution of a variety of galactic sources. We also show that the cross-correlation of the data sets from two interferometers could marginally lead to meaningful upper-limits on the degree of isotropy of the primordial gravitational wave background.

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Gravitational wave (GW) stochastic backgrounds generated in the early-Universe and produced by large populations of faint astrophysical sources are among the most interesting signals for GW laser interferometers. Their detection will provide a new arena for early-Universe cosmology and compact-object astronomy. The Laser Interferometer Space Antenna (LISA) [1] represents the key instrument in the low-frequency band of the spectrum ($\sim 10^{-5}\,\mathrm{Hz} - 10^{-1}\,\mathrm{Hz}$), complementary to Earth-based detectors, operating in the window $\sim 1\,\mathrm{Hz} - 10\,\mathrm{kHz}$. LISA could prove to be the most suitable device to search for, and study stochastic signals: (i) Several cosmological models predict a primordial GW stochastic background with a flat energy density spectrum $\Omega_{\rm gw}(f) \sim$ const (see [2], and references therein, for a thorough review): indeed the spectral density of the signal $S_h(f)$ generated at the detector output steeply increases at low frequencies as $S_h(f) \propto \Omega_{\rm gw}(f)/f^3$; (ii) Stochastic backgrounds generated by the incoherent super-position of GWs from a variety of short-period binary systems (including close white dwarf binaries, W UMa binaries, and neutron star binaries) both galactic [3–6] and extra-galactic [7,8] are *guaranteed* signals for LISA, with several components that dominate the instrumental noise in the frequency range $10^{-4}\,\mathrm{Hz} \lesssim f \lesssim 3\times 10^{-3}\,\mathrm{Hz}$.

The key issue in searching for stochastic backgrounds, and gaining insights into their production mechanism and the underlying physics, is to identify unambiguously the GW signal in one single LISA data stream. In fact, a stochastic background is a random process which is intrinsically indistinguishable from the detector noise. We have discussed this issue in [9], in the context of *perfectly isotropic* signals, showing the advantage of cross-correlation experiments involving two data sets with uncorrelated noise, and pointing out fundamental sensitivity limits. More recently it has been shown that, by estimating the LISA instrumental noise using the "symmetrized Sagnac observable" [10] it is possible to detect an isotropic stochastic background with a sensitivity close to the one that can be achieved through cross-correlation [11]. However, *several stochastic backgrounds are expected to be anisotropic*. In particular, the background generated by galactic sources carries a strong signature due both to the peripheral location of the solar system in the galaxy, and the spatial distribution of the sources. The primordial GW background, although expected to be intrinsically isotropic to a high degree, presents a clear dipole structure due to the proper motion of the galaxy with respect to the cosmological rest frame.

In this paper we show that the peculiar orbital configuration of LISA – the instrument baricenter follows an Heliocentric orbit with period $T = 1$ yr, and the detector plane, tilted by 60° with respect to the Ecliptic, counter-rotates with the same period T [1] – can be exploited to design a suitable data analysis scheme to detect and study anisotropic stochastic signals using the Michelson configuration. This possibility was earlier suggested in [12]. Here we carry out a rigorous analysis from the point of view of data analysis, following [15], and we apply our results to the relevant astrophysical and cosmological scenarios. We provide estimates for the optimal signal-to-noise ratio (SNR) as a function of the signal parameters, and we show that, by detecting an anisotropic background, it is indeed possible to gain key insights on the spatial distribution of short-period binary systems. We also discuss the possibility of obtaining meaningful upper-limits on the degree of isotropy of the primordial GW background, and comment on the improvement of the sensitivity and information extraction that would come from the correlation of the data streams of two separated and suitably oriented instruments.

We start by briefly reviewing the fundamental data analysis concepts; we assume the reader familiar with the literature on the detection of stochastic backgrounds in GW observations [13–15]. The GW background amplitude at any time t and position $\vec{x}$ can be written in the transverse-traceless gauge as

$$h_{ab}(\vec{x},t) = \sum_{A=+,\times} \int d\hat{\Omega} \int_{-\infty}^{\infty} df e^{2\pi i(ft-\hat{\Omega}\cdot\vec{x}/c)} \times e^{A}_{ab}(\hat{\Omega}) h_A(\hat{\Omega},f)\,; \tag{1}$$

e^A_{ab} $(a,b = 1,2,3)$ is the polarization tensor of the wave, $h_A(\hat{\Omega},f)$ are the two independent polarization amplitudes $(A = +,\times)$, and $\hat{\Omega}$ is the unit vector on the two-sphere along the wave's propagation direction. Here we consider stochastic signals that are Gaussian, unpolarized but *not* isotropic: $\langle h_A(f,\hat{\Omega})\rangle = 0$ and $\langle h_A(f,\hat{\Omega}) h^*_{A'}(f',\hat{\Omega}')\rangle = \delta_{A\,A'}\delta(f-f')\delta^2(\hat{\Omega}-\hat{\Omega}')H(|f|)P(\hat{\Omega})$. $\langle . \rangle$ stands for the ensemble average, and we have assumed that the frequency of the signal is uncorrelated with respect to the angular distribution. $H(|f|)$ and $P(\hat{\Omega})$ are related to the spectrum $\Omega_{\rm gw}(f) \equiv d\rho_{\rm gw}(f)/\rho_{\rm c} d\ln f$ by [15]

$$\Omega_{\rm gw}(f) = \frac{8\pi^2}{3\,H_0^2}|f|^3 H(|f|) \int_S d\hat{\Omega} P(\hat{\Omega})\,. \tag{2}$$

The one-sided power spectral density of the GW background simply reads $S_h(|f|) = H(|f|)/8\pi$. $P(\hat{\Omega})$ represents the contribution to $\Omega_{\rm gw}(f)$ from the different directions in the sky $\hat{\Omega}$; in the context of our analysis, it is useful to expand $P(\hat{\Omega})$ in terms of the multipole moments P_{lm} as $P(\hat{\Omega}) = \sum_{l,m} P_{l\,m} Y_l^m(\hat{\Omega})$. The signal strain at the interferometer output is $h(t) = D^{ab}(t)\,h_{ab}(\vec{x}(t),t)$, where $D_{ab} = [\hat{u}_a(t)\hat{u}_b(t) - \hat{v}_a(t)\hat{v}_b(t)]/2$ is the detector response tensor, and $\hat{u}_a$ and $\hat{v}_a$ are the unit vectors pointing, at each time, in the direction of the instrument arms; $\vec{x}(t)$ describes the trajectory of the interferometer centre of mass, and for LISA the explicit time-dependent expressions can be found in [9]. The detector output $o(t) = h(t) + n(t)$ is affected by the instrumental noise $n(t)$, that we assume to be Gaussian and stationary, with one-sided noise spectral density $S_n(f)$.

The key idea to detect (and study the properties of) an anisotropic GW stochastic background is to break up the data set(s) of length $T_{\rm obs}$ into several data chunks of much smaller size $\tau \ll T < T_{\rm obs}$, over which the detector location and orientation is essentially constant, and construct the new signal

$$S(t) = \int_{-\infty}^{+\infty} df\, \tilde{o}_j(f,t)\, \tilde{Q}(f)\, \tilde{o}^*_k(f,t)\,. \tag{3}$$

In Eq. (3) the indexes j and k label the instruments, and $\tilde{o}(f;t) = \int_{t-\tau/2}^{t+\tau/2} dt' e^{-2\pi i f t'} o(t')$ is the Fourier transform of the data chunk of length τ, centered around t. $S(t)$ is the correlation function suitably weighted by the optimal (real) filter function $\tilde{Q}(f)$ (to be determined in order to maximize the signal-to-noise ratio) [15]. One then looks for peaks in $S(t)$ at multiples of the LISA rotation frequency $1/T$: because $\tau \ll T$ the correlation will vary as LISA changes orientation. To fix ideas, we can set $\tau \sim 10^5$ sec and $T_{\rm obs} \sim 10^8$ sec. If one considers two separated instruments, an additional time scale appears: the light-travel-time between the instrument centre-of-mass ξ. For two detectors in Heliocentric orbit at distance D, $\xi \simeq 500\,(D/{\rm AU})$ sec. As most of the correlation between two detectors builds up for frequencies $f \lesssim 1/\xi$, there is significant signal for time-scales shorter than τ. Since the LISA motion is periodic, one can decompose the mean value of $S(t)$ as a Fourier series:

$$\langle S(t)\rangle = \sum_{-\infty}^{+\infty} e^{i\,2\pi m t/T}\langle S_m\rangle \tag{4}$$

where each harmonics S_m reads

$$S_m = \frac{1}{T_{\rm obs}} \int_0^{T_{\rm obs}} e^{-i\,2\pi m t/T} S(t)\,dt\,. \tag{5}$$

The amplitude of S_m represents the observable whose signal-to-noise ratio is $({\rm S/N})_m = \mu_m/\sigma_m$, where $\mu_m = |\langle S_m\rangle|$, and $\sigma^2_m = \langle S_m\, S_{-m}\rangle - |\langle S_m\rangle|^2$. It is clear that the noise contribution to the ensemble average $\langle\rangle$, and, as a consequence, the SNR of each harmonics, is different whether o_j and o_k share correlated noise or not. We will therefore analyze the two cases in turn.

We start by considering the LISA mission. In this case $o_j = o_k = o$. Evaluating the expectation values $\langle . \rangle$, and defining a suitable positive semi-definite scalar product [15], which allows us to derive the explicit expression of $\tilde{Q}(f)$, the SNR of the m-th harmonics becomes

$$\left(\frac{\rm S}{\rm N}\right)_m = T_{\rm obs}^{1/2}\,\Theta_m^{(0)} \left\{\int_0^\infty df\, \mathcal{J}^0(f)\right\}^{1/2}\,, \tag{6}$$

where

$$\mathcal{J}^{(0)}(f) \equiv \frac{4\,S_h(f)^2}{4\,\gamma_0^2\,S_h(f)^2 + 20\,\gamma_0\,S_h(f)\,S_n(f) + 25\,S_n(f)^2}\,, \tag{7}$$

and

$$\Theta_m^{(0)} = \left|\sum_{l=|m|}^{\infty} P_{lm}\gamma_{lm}^{(0)}\right|\,. \tag{8}$$

$\gamma_0 = 3/4$ is the overlap reduction function [13,14] for two co-located and co-aligned detectors, and $\gamma^{(0)}_{lm} = 5/(16\pi^2)\int_0^{2\pi} d\alpha \int d\hat{\Omega}\, Y_{lm}(\hat{\Omega}) \sum_A \left[D_{ab}(\alpha) e_A^{ab}(\hat{\Omega})\right]^2$ are the overlap reduction functions corresponding to the l-th multipole moment and the m-th harmonic. Notice that for co-located and co-aligned instruments γ_0 and $\gamma^{(0)}_{lm}$ do not depend on the frequency. It is straightforward to show that $\gamma^{(0)}_{lm} = 0$ for l odd and $l > 4$. *LISA is therefore sensitive only to quadrupole ($l = 2$) and octupole ($l = 4$) anisotropy, and is completely "blind" to the whole set of odd multipole moments.* This property affects both the signal detection and parameter estimation: LISA is not sensitive to the (usually strongest) dipole anisotropy, and

only two multipole moments can be exploited in order to extract information.

The function $\mathcal{J}^{(0)}(f)$ in (6) takes a particularly simple form in the two physically relevant cases: (i) If $h(t) \gg n(t)$ (*e.g.* for backgrounds generated by galactic binaries for $f \lesssim 3$ mHz [3,4]) then $\mathcal{J}^{(0)}(f) \simeq 1/\gamma_0^2$; (ii) For $h(t) \ll n(t)$ (the likely situation for primordial backgrounds [2]), $\mathcal{J}^{(0)}(f) \simeq [2S_h(f)/5S_n(f)]^2$. Using the two approximate expressions for $\mathcal{J}^{(0)}(f)$, and assuming that the main contribution to the SNR comes from a frequency band $\Delta f \sim f$, over which $S_n(f)$ and $S_h(f)$ are roughly constant, Eq. (6) can be cast, respectively, in the form

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_m \approx 4.2 \times 10^2\, \Theta_m^{(0)} \left(\frac{\eta}{10^5}\right)^{1/2} \tag{9}$$

and

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_m \approx 1.3 \times 10^2\, \Theta_m^{(0)} \left[\frac{h_{\mathrm{c}}(f)}{h_{\mathrm{rms}}(f)}\right]^2 \left(\frac{\eta}{10^5}\right)^{1/2}\,, \tag{10}$$

where $\eta \equiv T_{\mathrm{obs}}\,\Delta f$; as reference values we consider $\Delta f = 10^{-3}$ Hz and $T_{\mathrm{obs}} = 10^8$ sec. h_{c} and h_{rms} are the characteristic amplitude of the signal and the noise, respectively, per logarithmic frequency interval (see *e.g.* [9,14]). Notice that when $h_{\mathrm{c}} \ll h_{\mathrm{rms}}$, the SNR is reduced by the factor $[h_{\mathrm{c}}(f)/h_{\mathrm{rms}}(f)]^2 \ll 1$ with respect to the case where the (isotropic component of the) signal dominates the noise. Eq. (9) represents the sensitivity limit of LISA to anisotropic backgrounds: when $h_{\mathrm{c}}(f) \gtrsim h_{\mathrm{rms}}(f)$, the reduction of the instrumental noise does not improve the SNR.

In the LISA frequency band we expect a strong stochastic background generated by the galactic population of close binary systems, mainly white dwarf (WD) binaries [3–5]. WD binary systems are mainly located in the galactic disk, and the background that they generate is anisotropic for an observer in the solar system. In order to gain insight into the LISA ability of detecting such signature we model the WD distribution as being uniform within a cylinder of radius r_0 and height $2z_0$ (with $r_0 > z_0$); typical values are $r_0 \approx 5.5$ kpc and $z_0 \approx 0.3$ kpc [3]. By making this approximation the computation of P_{lm} and $\Theta_m^{(0)}$ simplifies considerably. For $S_h(f)$ and $S_n(f)$ we use the estimate provided in [3,4] and [1], respectively, whose analytical approximations are given by Eq. (3.1) and (2.6) of [9]. Eq. (9) yields:

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_1 \approx 20 \left(\frac{\eta}{10^5}\right)^{1/2}, \left(\frac{\mathrm{S}}{\mathrm{N}}\right)_2 \approx 60 \left(\frac{\eta}{10^5}\right)^{1/2}. \tag{11}$$

This result clearly shows that LISA can unambiguously identify the anisotropic component of the stochastic background generated by galactic WD binaries. By exploiting this signature one could also disentangle the galactic contribution from the extra-galactic one, which is isotropic to a large extent, being dominated by sources at $z \sim 1$ [8]. The large SNR at which detection can be made will enable us to extract valuable information about the distribution of binary systems in the galaxy. However only the harmonics with $m = 1$ and 2 will clearly stand above the noise; for $m > 2$ the SNR is negligible. We have checked that the former results are weakly sensitive on the approximations that we have made: Eq. (11) agrees within $\approx 15\%$ with the value obtained by the numerical integration of Eq. (6) using Eq. (7) over the entire sensitivity window and considering a more sophisticated source distribution. The galactic background from WD-WD binaries satisfies the condition $h_{\mathrm{c}}(f) \gg h_{\mathrm{rms}}(f)$ over the sensitivity window where most of the SNR is accumulated. If other galactic populations in the disk generate a stochastic background buried into the noise, LISA could in principle detect the signal, but at a SNR smaller than (11) by the factor $\sim [h_{\mathrm{c}}(f)/h_{\mathrm{rms}}(f)]^2$, which makes rather unlikely to discriminate the two populations.

LISA is very effective in detecting the anisotropy of signals from sources located in the disk; one might wonder whether populations characterized by a spherical distribution (such as MACHO binary systems in the Galactic halo [16]) would also produce a detectable anisotropy. We assume for sake of simplicity that the sources are uniformly distributed within a sphere of radius r_c. Depending on the value of r_c, whether it is smaller or greater than $r_{\mathrm{GC}} \simeq 8.5$ kpc, the distance of the solar system from the Galactic Centre, the sources produce a different degree of anisotropy. We consider first the case of a *bulge* distribution, where $r_c < r_{\mathrm{GC}}$. Assuming $h_{\mathrm{c}}(f) \gg h_{\mathrm{rms}}(f)$, and $r_c = 1$ kpc, Eq. (9) yields:

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_1 \approx 132 \left(\frac{\eta}{10^5}\right)^{1/2}, \left(\frac{\mathrm{S}}{\mathrm{N}}\right)_2 \approx 13 \left(\frac{\eta}{10^5}\right)^{1/2}. \tag{12}$$

We have checked that if $r_c = 0.5$ kpc, the SNR is essentially unaffected, and for $r_c = 7$ kpc the SNR becomes ≈ 78 and 7 for $m = 1$ and 2, respectively. If $h_{\mathrm{c}} \lesssim h_{\mathrm{rms}}$ the values in Eq. (12) are reduced by the factor $\approx 0.3\,[h_{\mathrm{c}}(f)/h_{\mathrm{rms}}(f)]^2$. For sources located in an extended *halo*, with $r_c > r_{\mathrm{GC}}$, the multipole moments scale as $P_{lm} \sim (r_{\mathrm{GC}}/r_c)^l$, and Eq. (9) yields:

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_1 \approx 17 \left(\frac{8.5\,\mathrm{kpc}}{r_c}\right)^2 \left(\frac{\eta}{10^5}\right)^{1/2},$$
$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)_2 \approx 10 \left(\frac{8.5\,\mathrm{kpc}}{r_c}\right)^2 \left(\frac{\eta}{10^5}\right)^{1/2}. \tag{13}$$

LISA can therefore provide information on a background generated by sources characterized by a spherical distribution. Eqs. (11), (12) and (13) show that LISA can indeed offer a radically new mean to explore the distribution of compact objects in the galaxy. However, the task of pin-pointing the exact structure of the population will be highly non-trivial [17].

Although the former results are encouraging, they also suggest that it is unlikely to be able to gain significant information on the much smaller degree of anisotropy produced by extra-galactic generated backgrounds, for which

$\Theta_m^{(0)} \ll 1$, even if $h_c \gg h_{\rm rms}$, see Eq. (9). *A fortiori* the study of the anisotropy of the primordial GW background is essentially ruled out. In principle one could attempt to detect the dipole anisotropy induced by the motion of our local frame with $v/c \sim 10^{-3}$ with respect to the Hubble flow. However LISA is not sensitive to $l = 1$, and one would have to rely on the next "visible" multipole moment ($l = 2$). For $l = 2$ we have $\Theta_m^{(0)} \lesssim 10^{-6}$, and it would require a ridiculous time of integration to achieve SNR > 1.

The tremendous sensitivity ensured by the space technology, and the clear advantage of flying two independent LISA instruments [9] – possibly with the option of achieving optimal sensitivity in the frequency band 0.1 Hz $\lesssim f \lesssim$ 1 Hz, which is likely free from astrophysically generated backgrounds – is stimulating further work regarding the new range of opportunities provided by cross-correlation experiments [17,18]. The main advantage of this configuration for the study of the background anisotropy is the full sensitivity to the entire set of multipole moments l: in general $\gamma_{lm}(f) \neq 0 \ \forall l$ [15]. The value and frequency dependence of $\gamma_{lm}(f)$ depend on the actual separation and relative orientation of the two instruments, and one could envisage to "tune" the configuration to some particular anisotropy structure. For two identical interferometers located at a distance D, the SNR reads (cfr. also [15])

$$\left(\frac{\rm S}{\rm N}\right)_m = \sqrt{T_{\rm obs}} \left\{ \int_0^\infty df\, \mathcal{J}(f) \Theta_m^2(f) \right\}^{1/2} , \qquad (14)$$

where $\mathcal{J}(f)$ and $\Theta_m(f)$ are the analogous of $\mathcal{J}^{(0)}(f)$ and $\Theta_m^{(0)}$, obtained by replacing γ_0 and $\gamma_{lm}^{(0)}$ with $\gamma(f)$ and $\gamma_{lm}(f)$, respectively.

We have estimated the SNR that one could achieve by cross-correlating the data sets of two identical LISAs, placed at a distance $D = 1$ AU, and for a few "random" orientations of the instruments. The key difference with respect to the LISA mission is that the odd multipole moments, in particular $l = 1$ and $l = 3$, become visible; as a consequence the harmonics with $m = 1$ will become stronger, and also the harmonic with $m = 3$ could be observable. As an example, for a disk distribution, one would achieve $({\rm S/N})_3 \gtrsim 3$; for bulge and extended halo distributions $({\rm S/N})_1$ would be a factor ≈ 2 and 10 higher than (12) and (13), respectively. This will impact on the ability of the instrument of gaining insight on the source distributions, and disentangling the contributions from the disk, the bulge and the halo. However this particular aspects deserve further investigation.

Cross-correlations would also make marginally possible to set limits on the anisotropy of the GW primordial background, thanks to the sensitivity to the dipole moment. Let us consider a primordial background with $P(\hat\Omega) = 1 + \beta_D \hat\Omega \cdot \hat\Omega_D + O(\beta_D^2)$, where the parameter β_D measures the degree of anisotropy and the unit vector $\hat\Omega_D$ defines the direction of maximum anisotropy. Assuming that the primordial spectrum overwhelms both the intrinsic noise and the astrophysical background (i.e. $h_{100}^2\,\Omega_{\rm gw} \gtrsim 10^{-10}$) the parameter β_D must satisfy $\beta_D > 8 \times 10^{-3}\ (T_{\rm obs}/10^8\,{\rm sec})^{-1/2}\ (\Delta f/10^{-3}\,{\rm Hz})^{-1/2}$ in order to detect the dipole component. The dipole anisotropy induced by the motion of the local reference frame with respect to the Hubble flow corresponds to $\beta_D \sim 10^{-3}$; therefore only an instrument with optimal sensitivity in the band $f \gtrsim 0.1$ Hz would be able to pick it up. This band presents also the considerable advantage of being completely transparent to primordial GW backgrounds [9]. Assuming that one can perfectly remove the induced dipole anisotropy from the data using independent information provided by CMB experiments, one could then be able to test an *intrinsic* statistical departure from isotropy compatible with the measurement of the quadrupole anisotropy of the CMB temperature [19].

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